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Modelling low order statistics of temperature increments in fully developed turbulence

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Abstract—This paper reports an experimental data analysis which clearly emphasizes the complex nature of the mechanisms governing the mixing of passive scalars such as temperature in fully developed turbulence. For that purpose, we compare our measurements of temperature increments and their probability density functions (pdfs) to theoretical predictions available in the literature for a scalar field evolving within a fully stationary developed dynamical turbulent field. The observed disagreements lead us to propose some improvements of the existing models. This, in fact, underlines the evolution through the scales of turbulence of the statistics of temperature increments, whose coupling with the velocity field appears to be an essential feature of the mixing process. © 1998 Elsevier Science Ltd. All rights reserved.

1. INTRODUCTION

The intermittent nature of small scales in fully developed turbulence has already been investigated quite widely over the last few decades. One of the main properties inferred from these investigations is that the small-scale velocity $\delta U(r) = U(x+r)-U(x)$, or temperature $\delta\theta(r)$ increments statistics, are very far from Gaussianity. Over the last 30 years, a large amount of work has been devoted either experimentally, theoretically or numerically, to the study of the impact of the energy transfer rate fluctuations upon the global statistical properties of turbulence. These works [1–4] have tried to establish the evolution, as a function of the order p, of the exponents of the power laws of high-order moments of $\delta U(r)$ or $\delta\theta(r)$, and to compare this evolution with theoretical predictions. Most of these theories are based on phenomenological ideas that describe the breaking of eddies in smaller eddies and quantify the energy distribution at each step of the cascade. Among these models, the more realistic ones are founded on log-normal probability [5], on fractality or multifractality [6, 7] or on more refined statistics [8, 9], but for a few years several studies have directly tried to predict the evolution of the pdfs through the scales of turbulence. In this framework, some studies [10, 11] have characterized quite precisely the pdfs of the velocity increments δU, with special care of the behavior of their wings. Nevertheless, due to peculiar mathematical properties, and also to recent efforts and progress in combustion modelling [12], the most promising effects now concern the scalar field statistical properties [13]. For instance, an exact result, which gives the shape of the pdf of a scalar field mixed in a stationary process, has recently been established [14]. More recently, Vaienti *et al.* [15] have developed a theory that leads to a partial differential equation for the evolution of the pdf of temperature increments, through the scales r of turbulence. In a similar way, Valino *et al.* [16] obtained an exact equation that predicts the shapes of the pdfs of a passive scalar as a function of the flatness coefficient of its distribution. This work will be referred to as VDR in the following. It is in this framework that we propose the present study, which deals with an extension of the results of the VDR model to temperature increments.

First, we will recall the main results from this VDR model, then we will present the experimental conditions of our study. Finally, we will compare the experimental shapes of the temperature increments pdfs with those obtained by VDR and with a new extended model that we propose.

2. MODELLING THE PROBABILITY DENSITY FUNCTION

The model developed by VDR is originated from the following equations (1a) and (1b) [12] which describe the time evolution of the pdf of a scalar θ advected by homogeneous and isotropic turbulence.

$$\frac{\partial P}{\partial t} = -\frac{\partial}{\partial \chi}[(R+\varepsilon_X\chi)P] \quad \text{with } R(\chi,t) = \langle \kappa\nabla^2 X | X=\chi\rangle \qquad (1a,b)$$

$$\frac{\partial P}{\partial t} = -\frac{\partial}{\partial \chi}\left[\frac{\partial}{\partial \chi}(QP)+\varepsilon_X\chi P\right] \quad \text{with } Q(\chi,t) = \langle \kappa(\nabla X)^2 | X=\chi\rangle \qquad (2a,b)$$

where $\varepsilon_X = \kappa\langle(\nabla X)^2\rangle$ is the mean dissipation rate and

NOMENCLATURE

C_1/ε_X normalization condition of $P(\chi)$
C_2 variable linked to F by $C_2 = (F-3)/2F$ and to α and β by $C_2 = \beta/\alpha$
F fourth-order dimensionless moment of P
$P(\chi)$ probability density function of the variable χ
$Q(\chi)$ conditional dissipation rate of $\delta\theta$
$R(\chi)$ conditional diffusion rate of $\delta\theta$
r separation.

Greek symbols
α, β in our present model $Q(\chi)/\varepsilon_X = \alpha+\beta\chi^2$ with $\alpha+\beta = 1$
γ in our present model $R(\chi)/\varepsilon_X = -\gamma\chi$
δ boundary layer thickness
δU velocity increment $\delta U(r) = U(r+x)-U(x)$
$\delta\theta$ temperature increment $\delta\theta(r) = \theta(r+x)-\theta(x)$
ε_X mean dissipation rate
η Kolmogorov length scale
$1/\tau$ cut-off variable
χ normalized fluctuations of temperature increment.

$$X = \frac{\theta}{\sqrt{\langle\theta^2\rangle}}$$

and where $\langle\rangle$ represents a statistical averaging.

Assuming the existence of a quasistationary asymptotic stage, VDR generalizes the results of Pope and Ching [14], which predict, in a stationary regime, the shape of the pdf as a function of two conditional averages R and Q defined by relations (1b) and (2b). Indeed, an identification of equations (1a) and (2a) yields Pope and Ching's result:

$$P(\chi, t) = \frac{C_1}{Q(\chi, t)} \exp\left[\int \frac{R(X, t)}{Q(X, t)} \mathrm{d}X\right]. \tag{3}$$

Then, a quasistationarity hypothesis [12] implies the following simple form for the conditional expectation $R(\chi, t)$:

$$\frac{R(\chi, t)}{\varepsilon_X} = -\chi. \tag{4}$$

This specific linear function was also proposed by Ching [17] and by Pope and Ching [14]. Then, on the basis of numerical works [18], and also because of experimental evidences, the following parabolic shape is proposed for $Q(\chi, t)$:

$$\frac{Q(\chi, t)}{\varepsilon_X} = \frac{1+C_2\chi^2}{1+C_2}. \tag{5}$$

When multiplying one or the other of equations (1a) and (2a) by χ^{2n}, an equation linking the $(2n)$th-order moment to the $(2n-2)$th moment is obtained:

$$\frac{\mathrm{d}\overline{\chi^{2n}}}{\mathrm{d}t} = -2n\cdot(2n-1)\frac{\overline{\chi^{2n-2}\cdot\varepsilon_X}}{\varepsilon_X} + 2n\cdot\overline{\chi^{2n}}. \tag{6a}$$

It is then possible to link the coefficient C_2 through equation (6a), written for the fourth-order moment, to the flatness factor in the following manner:

$$C_2 = \frac{F-3}{2F}. \tag{6b}$$

Let us specify here that normalization conditions applied to the function Q and to the zeroth- and second-order moments of the pdf, imply that the slope of R is equal to -1, and that the sum of the coefficients $(1/(1+C_2))$ and $(C_2/(1+C_2))$ is equal to 1. Therefore, VDR calculates the shapes of the pdfs where the flatness factor F is a parameter:

for $1 < F < 3$

$$P(\chi, t) = \frac{F-1}{\sqrt{\pi}}\left(\frac{3-F}{2F}\right)^{3/2} \frac{\Gamma\left(\frac{6-F}{3-F}\right)}{\Gamma\left(\frac{3+F}{6-2F}\right)} \times \left(1+\frac{F-3}{2F}\chi^2\right)^{\frac{9-5F}{2F-6}}$$

for $F = 3$

$$P(\chi, t) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{\chi^2}{2}\right)$$

for $F > 3$

$$P(\chi, t) = \frac{F-1}{\sqrt{\pi}}\left(\frac{F-3}{2F}\right)^{3/2} \frac{\Gamma\left(\frac{3F-3}{2F-6}\right)}{\Gamma\left(\frac{3}{F-3}\right)} \times \left(1+\frac{F-3}{2F}\chi^2\right)^{\frac{9-5F}{2F-6}} \tag{7}$$

where Γ is the Euler function of the second kind.

The main goal of our work is to extend this model to temperature increments for which statistics are strongly non-Gaussian for small scales r, and for which we may think that the mixing process linked to vortex stretching may differ from a simple homogeneous mixing. In particular, it is worth noticing that the advection term of the energy conservation equation, which couples the thermal and dynamical fields, does not cancel out any more from the analysis, as is the case when the scalar field (and not its increment) is considered [15]. However, we will investigate the relevance of this kind of modelling to the present stationary case for which pdfs depend on the scale r.

3. EXPERIMENTAL CONDITIONS

Measurements are performed in the turbulent boundary layer developed on the working section floor of a low-speed wind tunnel with a cross-section 0.56×0.56 m^2. The wall is heated from the beginning of the layer to a constant temperature in such a manner that the temperature is acting as a passive scalar. At the measuring station ($x = 3.7$ m), the free stream velocity is 12 m s^{-1}, the boundary layer thickness is $\delta = 62$ mm, and the associated Reynolds number and Grashof number are, respectively, 50 000 and 350 000. At the position considered here ($y^+ = 310$, i.e. $y/\delta = 0.16$), the Kolmogorov length-scale η is 0.13 mm and the Reynolds number calculated with the Taylor micro-scale is about 180.

Simultaneous longitudinal velocity and temperature measurements are performed with a pair of parallel etched-Wollaston wires: the upstream cold wire ($l \sim 1$ mm, $d = 0.6$ μm; constant current, $I = 0.15$ mA) is separated by a distance of 0.4 mm from the hot wire ($l \sim 0.8$ mm, $d = 5$ μm; constant temperature, overheat of 0.7). The velocity and temperature increments are deduced from the temporal records by transforming a time-separation into a space-separation thanks to Taylor's hypothesis. The robustness of these measurements to study the statistics of temperature increments, even at small separation, has already been demonstrated [19]: the homo-

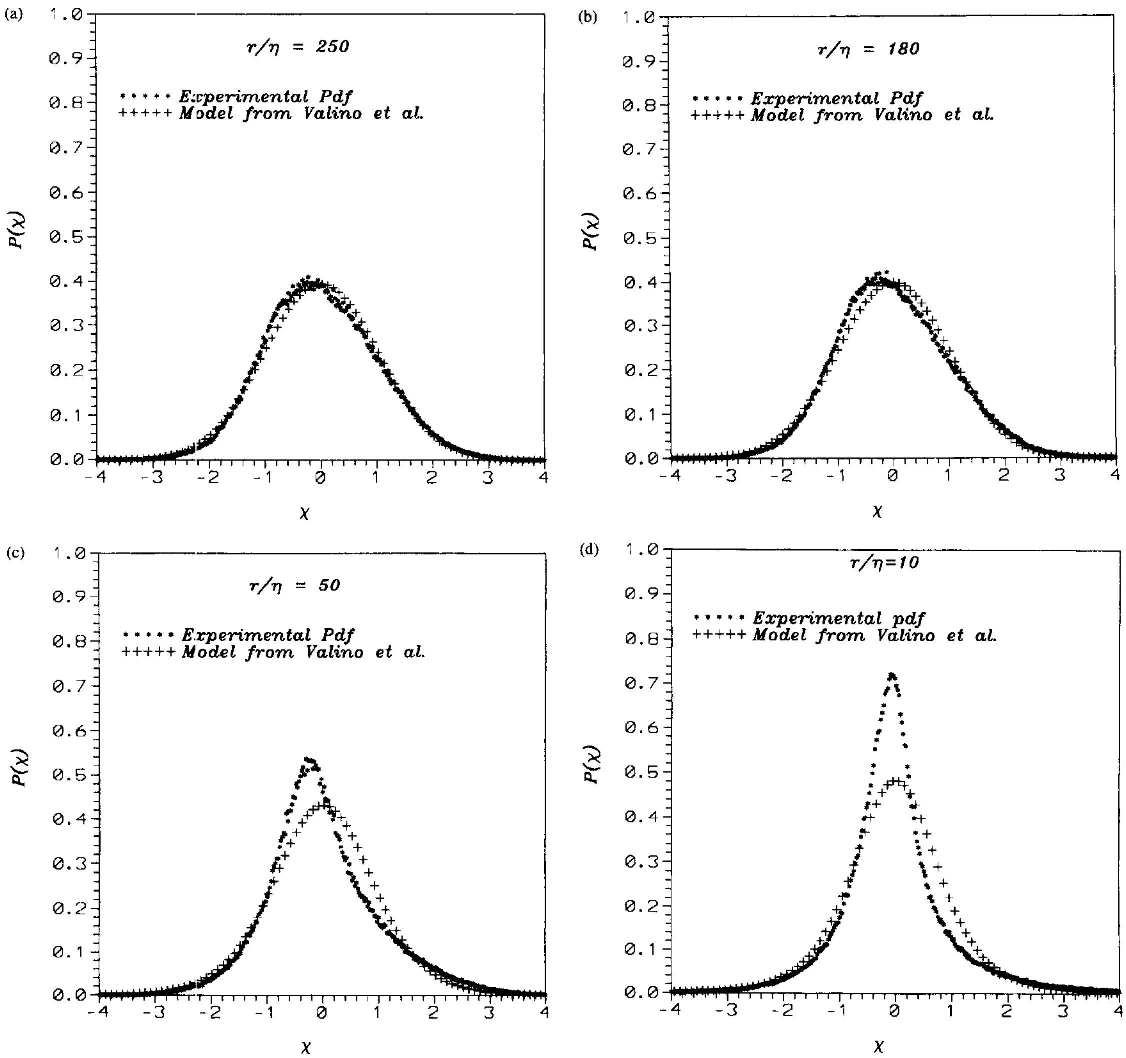

Fig. 1. Evolution of the measured and predicted pdfs P of temperature increments with the separation (a) $r/\eta = 250$; (b) $r/\eta = 180$; (c) $r/\eta = 50$; (d) $r/\eta = 10$.

geneous and isotropic nature of the flow is confirmed by this study. We know for instance that the inertial zone covers the range of scales from 30 η to 150 η. Notice that systematic tests have been necessary to obtain correct results for the smallest scales, for which the correlation coefficient between δU and $\delta\theta$ must be zero, because of isotropy. In particular, all noises, which maintain quite a strong correlation for the separations smaller than 10 η, have been minimized. The sampling frequency of 37.5 kHz satisfies the Shannon criterion and records of 512 000 points (or of double) have been analysed, assuring the good convergence of the different results reported hereafter.

4. THE VDR MODEL AND ITS EXTENSION APPLIED TO TEMPERATURE INCREMENTS

As presented by Ould-Rouis et al. [19], the flatness factors of $\delta\theta$ differ greatly from their Gaussian value for the scales corresponding to the inertial and dissipative zones ($r/\eta < 150$). Therefore, such experimental data seem to be particularly well adapted to test an extension of VDR model to temperature increments. Let us note, however, that the asymmetry of the pdf P will be neglected in the following as the theoretical predictions use vanishing odd moments. Figure 1 presents experimental pdfs measured for four values of the separation r. Each of these figures compares the experimental shape with the VDR model estimated with the experimental flatness F. For $r/\eta = 250$ or 180, i.e. when F does not deviate too much from 3, the theory reproduces in a satisfactory manner the features of the pdfs. As soon as the scale r is smaller than $r/\eta = 100$, the prediction deviates strongly from the measurements. Figure 2 shows that although these distributions possess the same flatness, their maxima differ strongly. In fact, a detailed analysis of the mathematical conditions imposed on the pdf, proves that the maxima predicted by VDR always remain lower than 0.5 for the considered experimental

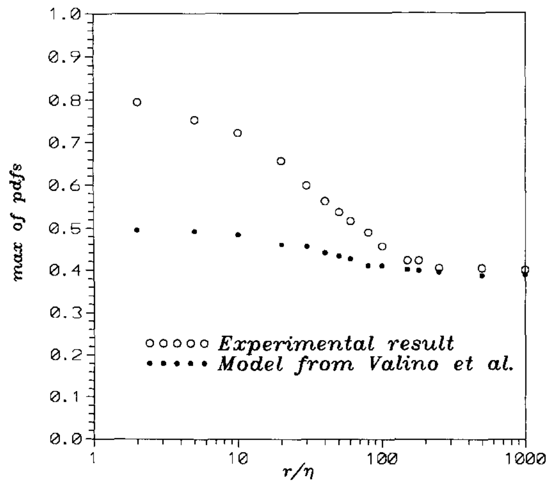

Fig. 2. Evolution of the maxima of temperature increments pdfs with the separation r/η.

flatness factor, whereas the experimental pdf maxima reach 0.75 at small scales. The origin of these disagreements comes, in part, from the model applied to the conditional expectations $R(\chi)$ and $Q(\chi)$. As shown in Fig. 3, the slope of the line $R(\chi)/\varepsilon_X$ evolves as a function of the separation r/η, whereas the model predicts it should remain equal to -1. Let us specify that this result is opposite to the findings of Ching [17] obtained in a Rayleigh–Bénard convection experiment (the scalar is not passive in this case, at least for the largest scales) and where $R(\chi)/\varepsilon_X$ stays close to a line of slope -1, even at very small scales. We note, however, that in a statistical analysis of the results of a numerical simulation of the Lorenz model [14], $R(\chi)/\varepsilon_X$ has a non-trivial shape quite different from a line of slope -1. Therefore, our results obtained in a slightly heated turbulent boundary layer suggests that it is necessary to relax the condition on the slope γ of R, by letting it move with r/η. Guided by our experiments, we choose:

$$\frac{R(\chi)}{\varepsilon_X} = -\gamma(r)\chi \tag{8a}$$

and

$$\frac{Q(\chi)}{\varepsilon_X} = \alpha(r)+\beta(r)\chi^2. \tag{8b}$$

However, as explained in Section 2 the VDR model implies (because of normalization conditions of Q and of zeroth- and second-order moments) that: $\alpha+\beta = 1$ and $\gamma = \alpha+\beta$. Thus, we need a new and additional parameter to relax the condition $\gamma = 1$. This new degree-of-freedom will come from the model applied to Q. As can be seen in Fig. 4, the conditional expectation Q possesses local maxima for large values of χ, followed by a decrease towards zero for even larger values (this has also been confirmed by a private communication of Aider and Wesfreid [20]). We then propose to model the function $Q(\chi)/\varepsilon_X$ by the parabola $\alpha+\beta\chi^2$, but only inside the interval $[-1/\tau, 1/\tau]$ and by the zero function outside this interval. In the same manner, all the functions and integrals will be considered equal to zero outside the same interval. Figure 5(a) shows the evolution with r of the coefficients α and β. The coefficient α has been calculated by a least-square interpolation method applied to the experimental function $Q(\chi)/\varepsilon_X$ under the constraint $\alpha+\beta = 1$. We note that the inverse calculation, with first the determination of β by interpolation and then deducing α by normalization conditions, gives the same results. In the same manner, Fig. 5(b) shows the variation of the coefficient, γ, which is calculated by a linear regression on the experimental functions $R(\chi)/\varepsilon_X$. Figure 6(a), (b) presents these evolutions vs the flatness factor and they are also compared to the predictions of the model. We observe that the model predictions deviate from the observations for small scales of turbulence, i.e. as soon as the flatness is significantly larger than 3.

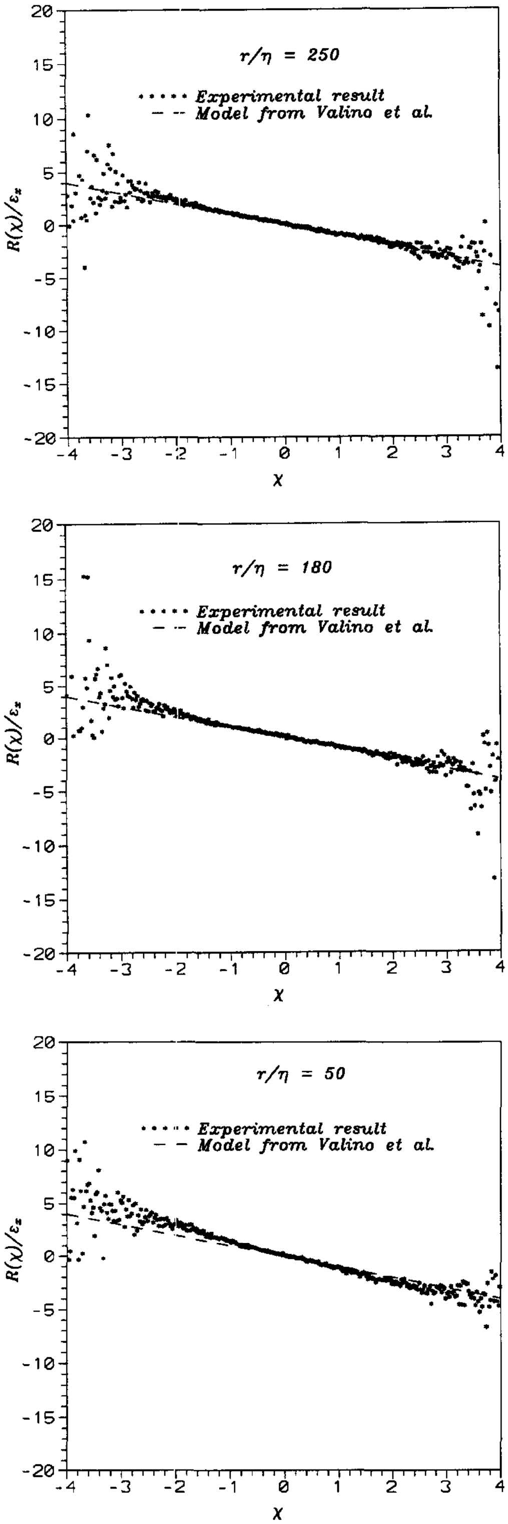

Fig. 3. Evolution of the conditional expectations R for different separations r/η (notice that the scales in graphs are very different): (a) $r/\eta = 250$; (b) $r/\eta = 180$; (c) $r/\eta = 50$; (d) $r/\eta = 10$.

In order to take into account the strong evolution of pdfs vs the turbulence scales, we will relax the condition on the slope of the function $R(\chi)$ by the use

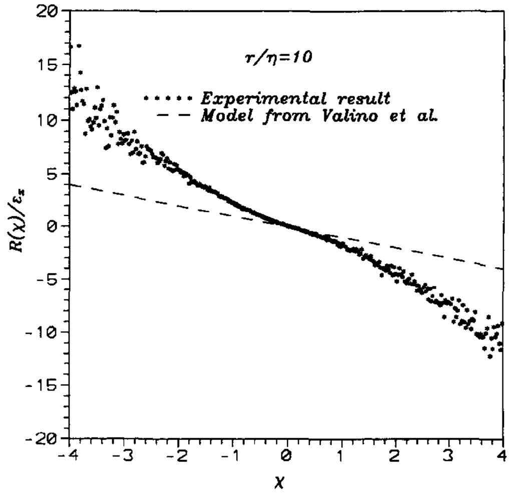

Fig. 3—*continued.*

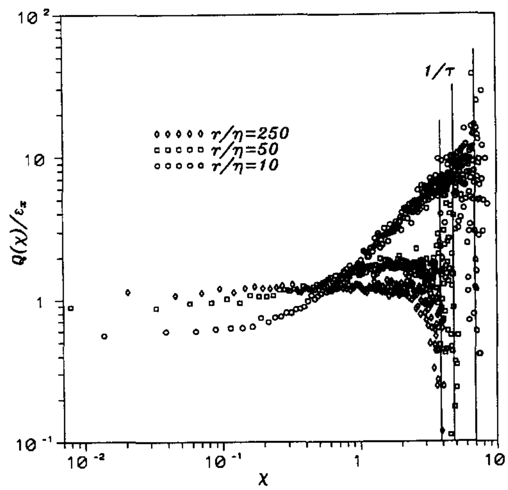

Fig. 4. The function Q for different separations r/η.

of the parameter τ. Notice, however, that the previous model will be recovered when τ tends towards zero.

Using the exact result of Pope and Ching [14] [equation (3)], and the models for the conditional expectations $R(\chi)/\varepsilon_X = -\gamma\chi$ and $Q(\chi)/\varepsilon_X = \alpha+\beta\chi^2$ on the reduced interval, it is possible to obtain an analytic form for the pdf $P(\chi)$ in the range $[-1/\tau, 1/\tau]$:

$$P(\chi) = \frac{(\alpha+\beta\chi^2)^{-\left(1+\frac{\gamma}{2\beta}\right)}}{2\int_0^{1/\tau}(\alpha+\beta X^2)^{-\left(1+\frac{\gamma}{2\beta}\right)}\,\mathrm{d}X}. \tag{9}$$

Unfortunately, the involved integral cannot be expressed as explicit analytical formula except for integer values of the exponent γ/β. Therefore, approximating γ/β by $\mathrm{Int}(\gamma/\beta)$, where Int means the nearest integer of the experimental value γ/β at the considered scale, we obtain the following analytical expressions

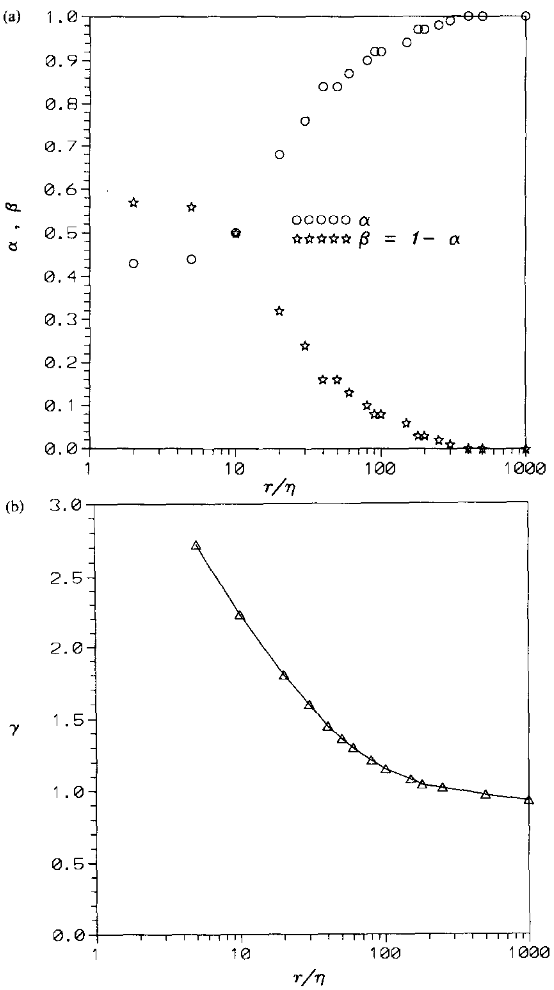

Fig. 5. Evolution of the coefficients α and β of Q (a), and of the slope γ of R (b), as a function of r/η.

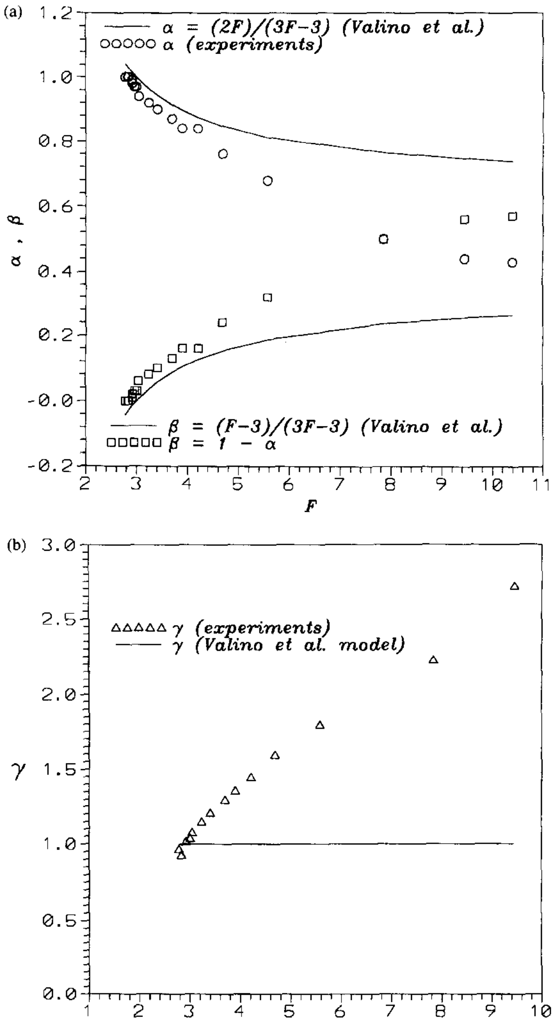

Fig. 6. Evolution of the coefficients α and β of Q (a), and of the slope γ of R (b), as a function of the flatness factor F.

for P, where α, β, χ and τ are coefficients determined from the experiments and γ/β is replaced by $\mathrm{Int}(\gamma/\beta)$: for γ/β odd, greater than 3

$$P(\chi) = \frac{\alpha^{\gamma/2\beta+1/2}(\alpha\tau^2+\beta)^{\gamma/2\beta}(\alpha+\beta X^2)^{-(1+\gamma/2\beta)}}{2\sum_{n=0}^{\frac{\gamma}{2\beta}-\frac{1}{2}} A_n \tau^{2n}\alpha^n\beta^{\gamma/2\beta-1/2-n}} \quad (10a)$$

with

$$A_{\frac{\gamma}{2\beta}-\frac{1}{2}} = 1 \quad \text{and} \quad A_n = \frac{A_{n+1}}{\left(\frac{1+\frac{\gamma}{2\beta}}{n+1}-1\right)}$$

for γ/β even, greater than 4

$$P(\chi) =$$

$$\alpha^{\gamma/\beta+1/2}\sqrt{\beta}\cdot(\alpha\tau^2+\beta)^{\gamma/2\beta}\cdot(\alpha+\beta\chi^2)^{-\left(1+\frac{\gamma}{2\beta}\right)} \Bigg/$$

$$2\left[\sum_{n=0}^{\frac{\gamma}{2\beta}-1}\sum_{i=n}^{\frac{\gamma}{2\beta}-1} C_n^i A'_i \alpha^{\gamma/\beta-n-1/2}\beta^{n+1/2}\tau^{\gamma/\beta-2n-1} + A'_{\gamma/2\beta-1}\arctan\left(\frac{\sqrt{\beta}}{\tau\sqrt{\alpha}}\right)\sum_{k=0}^{\frac{\gamma}{2\beta}} C_k^{\gamma/2\beta}\alpha^{\gamma/\beta-k}\beta^k\tau^{\gamma/\beta-2k}\right] \quad (10b)$$

with

$$A'_0 = \frac{\beta}{\gamma} \quad \text{and}\; A'_{i+1} = \frac{\gamma-\beta(1+2i)}{\gamma-2\beta(1+i)}A'_i.$$

In fact, the coefficients α, β, γ and τ are linked together by the normalization conditions applied on P and on its moments. This is precisely what VDR applied in order to link the coefficients of their model to the flatness factors. In our case, a similar calculation on the second-order moment using the normalization condition of the pdf leads to:

for γ/β odd, greater than 3

$$\frac{\tau^{\frac{\gamma}{\beta}-1}}{\alpha} + \sum_{n=0}^{\frac{\gamma}{2\beta}-\frac{3}{2}} A_n \tau^{2n} \alpha^{n-\gamma/2\beta-1/2} \beta^{\gamma/2\beta-1/2-n} \times \left[1-\frac{\alpha}{\gamma-\beta}\right] = 0 \quad (11a)$$

for γ/β even, greater than 4

$$\sum_{n=0}^{\frac{\gamma}{2\beta}-1}\sum_{i=n}^{\frac{\gamma}{2\beta}-1} C_n^i \alpha^{-n}\beta^n \tau^{\gamma/\beta-2n-1}\left(\frac{A_i'}{\alpha}-\frac{B_i'}{\beta}\right) + \arctan\left(\frac{\sqrt{\beta}}{\tau\sqrt{\alpha}}\right) \sum_{k=0}^{\frac{\gamma}{2\beta}} C_k^{\gamma/2\beta}\,\alpha^{-k+1/2}\beta^{k-1/2}\tau^{\gamma/\beta-2k} \times \left(\frac{A'_{\gamma/2\beta-1}}{\alpha}-\frac{B'_{\gamma/2\beta-1}}{\beta}\right)=0.$$

$$\text{where} \quad B_0' = -A_0' \quad \text{and} \quad B_i' = \frac{A_i'}{\frac{\gamma}{\beta}-1}. \quad (11b)$$

These expressions are quite untractable, but a particular solution can be obtained for $\tau = 0$. It can be verified that the condition $\gamma = \alpha+\beta = 1$ is recovered, and thus confirms the validity of our calculations. However, we are not able to close the problem with the use of flatness factors as done by VDR. The influence of the parameter τ on the pdf maxima is small. On the contrary, it acts strongly on the flatness value and it evolves from three times the rms value at large scales to eight times, at small scales.

Expressions (10a) and (10b) allow the explicit calculation of the pdfs; they are drawn on Fig. 7. It is easy to see that the agreement between the experimental and modelled results is good, as far as we neglect the asymmetry of the experimental pdfs. For very small scales, however, it can be seen that our

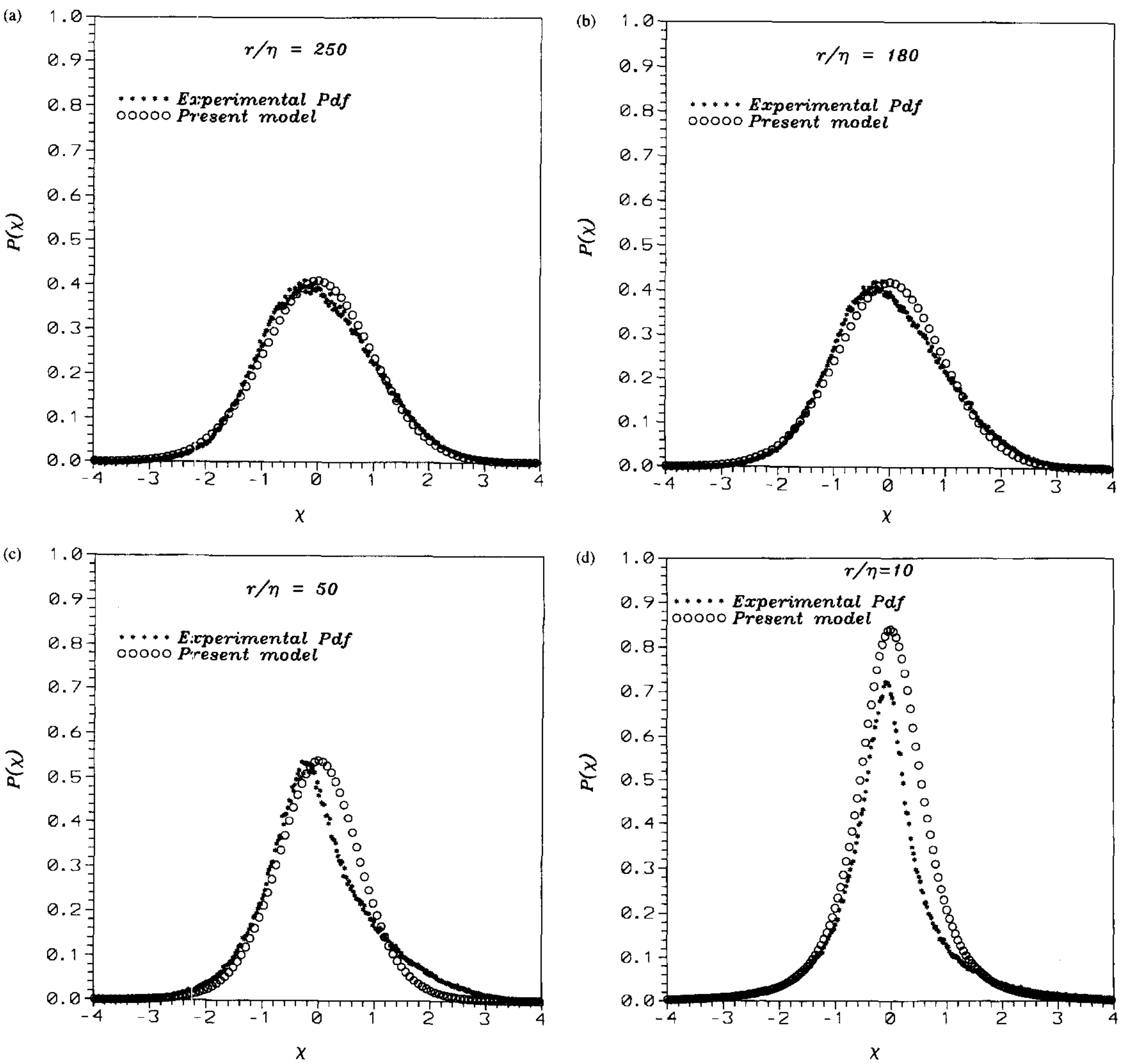

Fig. 7. Evolution of the predicted (new model) and experimental pdfs of temperature increments with the separation: (a) $r/\eta = 250$; (*b*) $r/\eta = 180$; (c) $r/\eta = 50$; (d) $r/\eta = 10$.

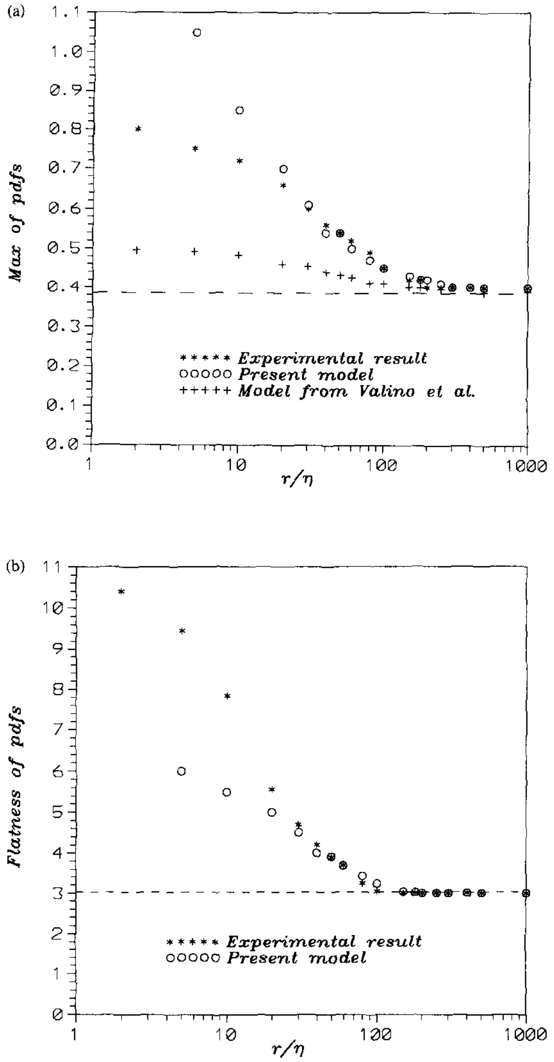

Fig. 8. Evolution of the maxima (a) and of the flatness (b) of the pdfs of temperature increments (experiments and models) with the separation r/η.

model predicts too large maxima. This disagreement may result from a too naive approximation for the function $R(\chi)$ which can present, for the smallest scales, a small wave in the vicinity of $\chi = 0$. A more sophisticated model which could take into account a smaller slope of R at $\chi = 0$ would induce a better prediction for the pdf maxima. Note that these observations are not surprising since the concerned small scales correspond to the dissipative zone of turbulence for which new transfer mechanisms are involved. Unfortunately, refining the function $R(\chi)$ results in untractable analytical developments. However, Fig. 8 presents the evolution of the flatness factor and of the maxima of the pdf vs the scale r. Comparison with the results of the VDR model shows indeed the significant improvement resulting from our extended model.

5. CONCLUSION

We have shown in this study that the Valino *et al.* model (VDR) [16] needs to be completed when it is no more used to study the temperature field, but its increments. We propose an extension which is able to correct some of the encountered disagreements. In particular, our analysis is based on the exact result of Pope and Ching [14] and on a proper estimation and modelling of the two conditional expectations $R(\chi)$ and $Q(\chi)$. In order to relax the very restrictive normalization conditions of the VDR model, two new parameters τ and γ are introduced. $-1/\tau$ and $1/\tau$ are the boundaries of the support of the experimentally measured functions $R(\chi)$, $Q(\chi)$ and $P(\chi)$ and γ is the slope of the function $R(\chi)$. Due to the complexity of the analytical calculations, we are not able (as it was done by VDR) to express the parameters of our model as explicit functions of the flatness factor. However, numerical estimations of the involved integrals permit to obtain new expressions for the pdfs of scalar increments which correctly predict the experimental trends as long as the considered turbulent scales are not too small ($r/\eta > 20$).

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